JOHNS HOPKINS
APPLIED PHYSICS LABORATORY

# Optimizing Quantum Annealing Performance via Quantum Control

## Qubits 2018: D-Wave Users Conference

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The Johns Hopkins University Applied Physics Laboratory

# Overview

**Problem**

Develop a method to optimize control schedules for *general* adiabatic quantum computation (AQC) algorithms that is

- Scalable/Efficient:  Convergence rates that do not depend on system size
- Practical: does not require knowledge of energy spectrum or computational solution
- Robust: robust to system uncertainty, e.g., noise

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**Motivation**

- Optimized control *can* facilitate computational speedup
  - Time-optimal controls for Grover's search algorithm (Rolad, Cerf PRA 2002, Rezakhani et al. PRL 2009)
  - Boundary cancellation methods (Rezakhani et. al. PRA 2011)
- Many techniques are not practical
  - Require knowledge of the instantaneous energy spectrum (Zeng et al JPA 2016)
  - Require knowledge of the computational solution (Brif at. Al NJP 2014)
  - Not robust to system uncertainty (Roland, Cerf PRA 2002, Rezakhani et al. PRL 2009)

# Quantum Control

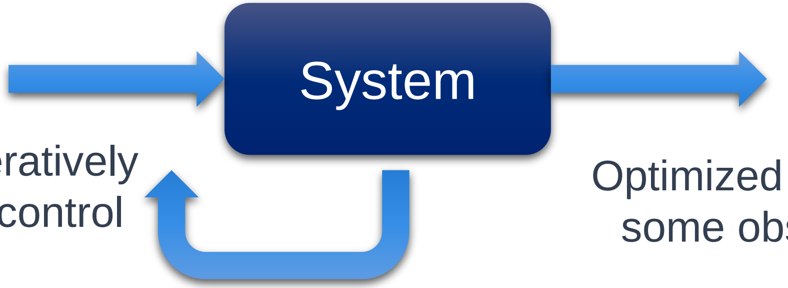
**Objective**

Perform particular quantum operation with high fidelity, potentially while simultaneously mitigating the effects of unwanted environment interactions

### Open Loop Control


- Relies on system model
- May/may not be robust to uncertainty
- Example: optimal control, robust control

### Closed Loop Control

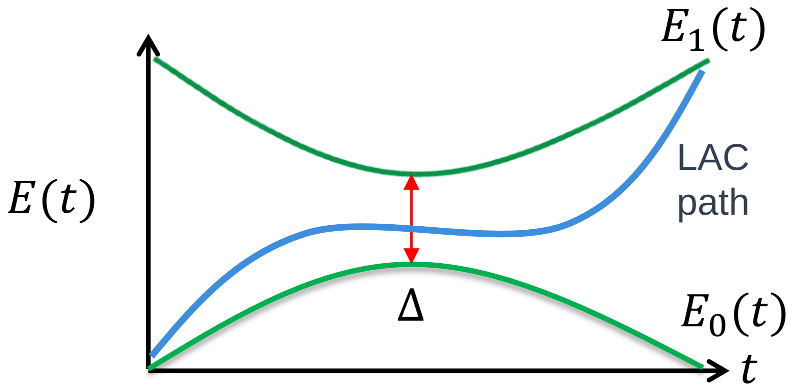



- Inherently robust to system uncertainty
- Requires intermediate or terminal measurement of an observable

# Quantum Control for Adiabatic Quantum Computation

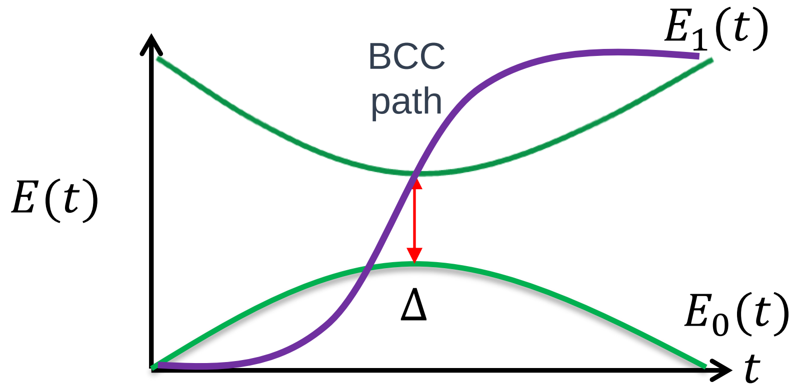
## Local adiabatic control (LAC)

- Relies on "instantaneous adiabatic theorem"
  - satisfy the adiabatic condition at each instance in time
- Minimizes the time needed to reach the adiabatic regime based on the rate of change of the evolution




## Boundary cancellation control (BCC)

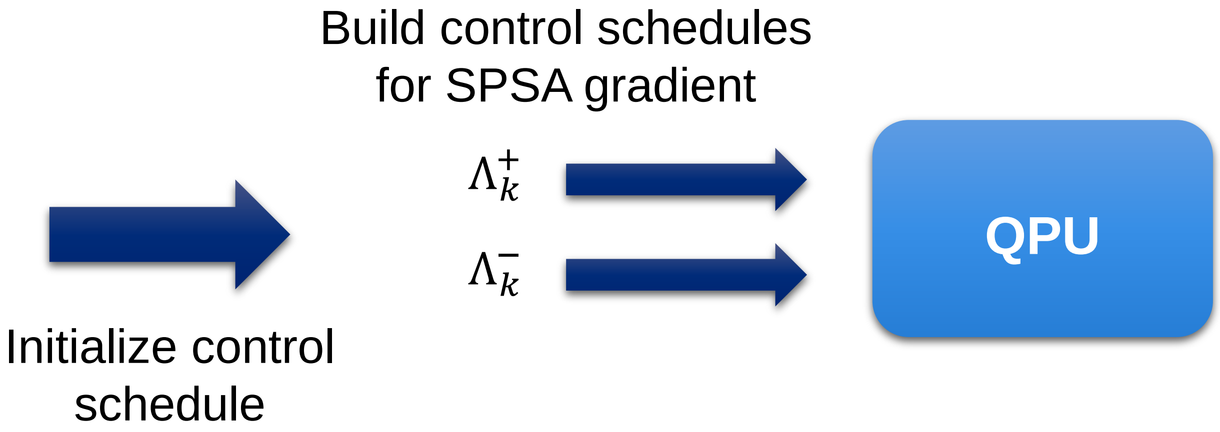
- Relies on "final time adiabatic theorem"
  - Minimizes error in the adiabatic approximation
  - Polynomial error improvement of LAC by setting the first $n-1$ derivatives of the Hamiltonian to zero at the boundaries



# Closed Loop Control Protocol

**Closed Loop Optimized Adiabatic Quantum Control (CLOAQC)**

*In situ* control protocol designed to minimize system energy via Simultaneous Perturbation Stochastic Approximation (SPSA) optimization (Spall 1992)



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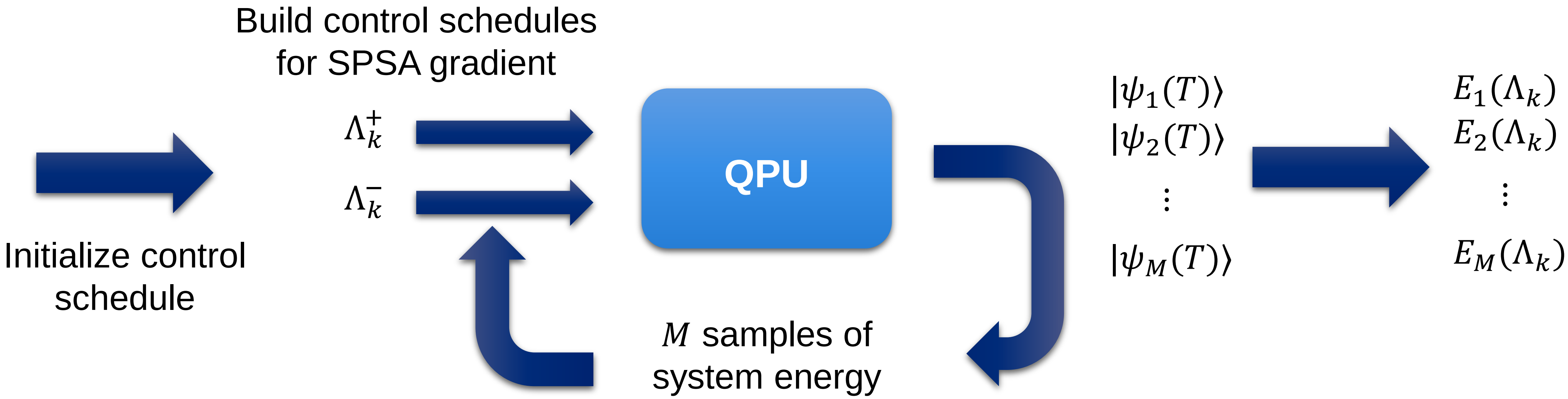

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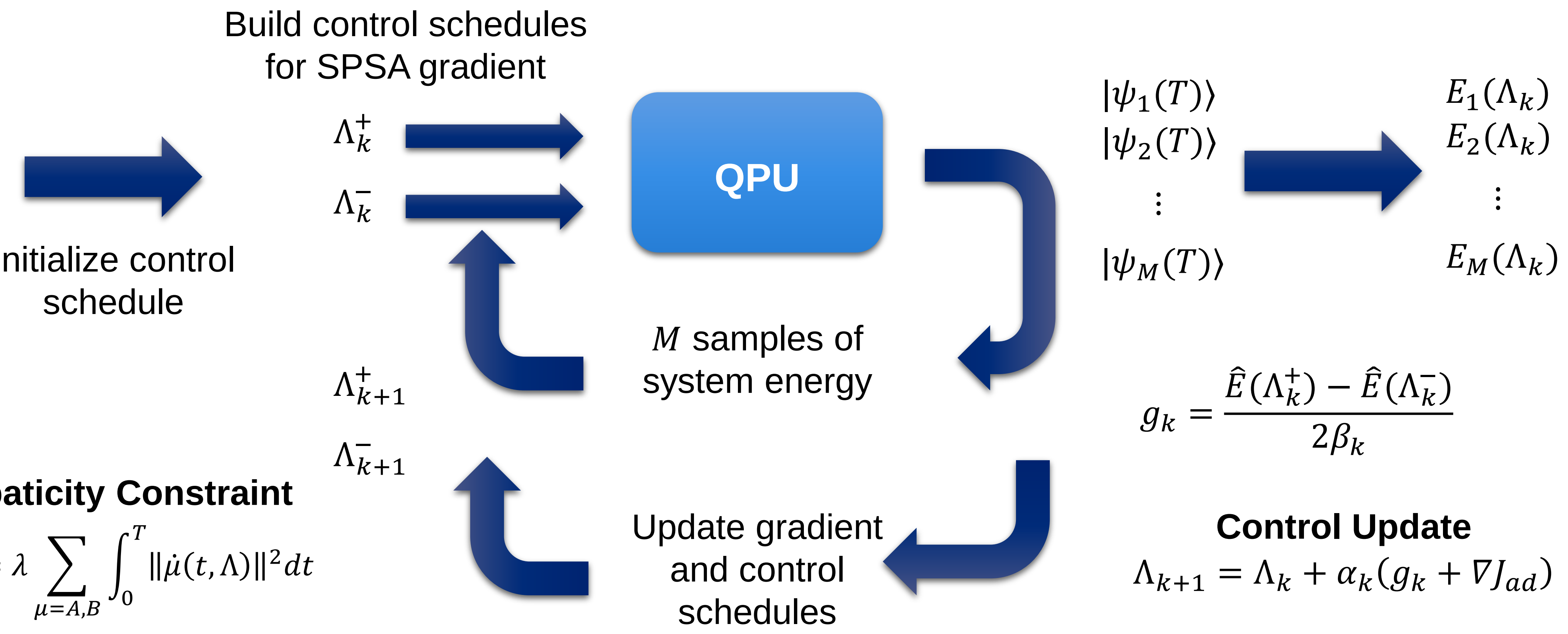


**Adiabaticity Constraint**

$$J_{ad}(\Lambda) = \lambda \sum_{\mu=A,B} \int_0^T \|\dot{\mu}(t,\Lambda)\|^2 dt$$

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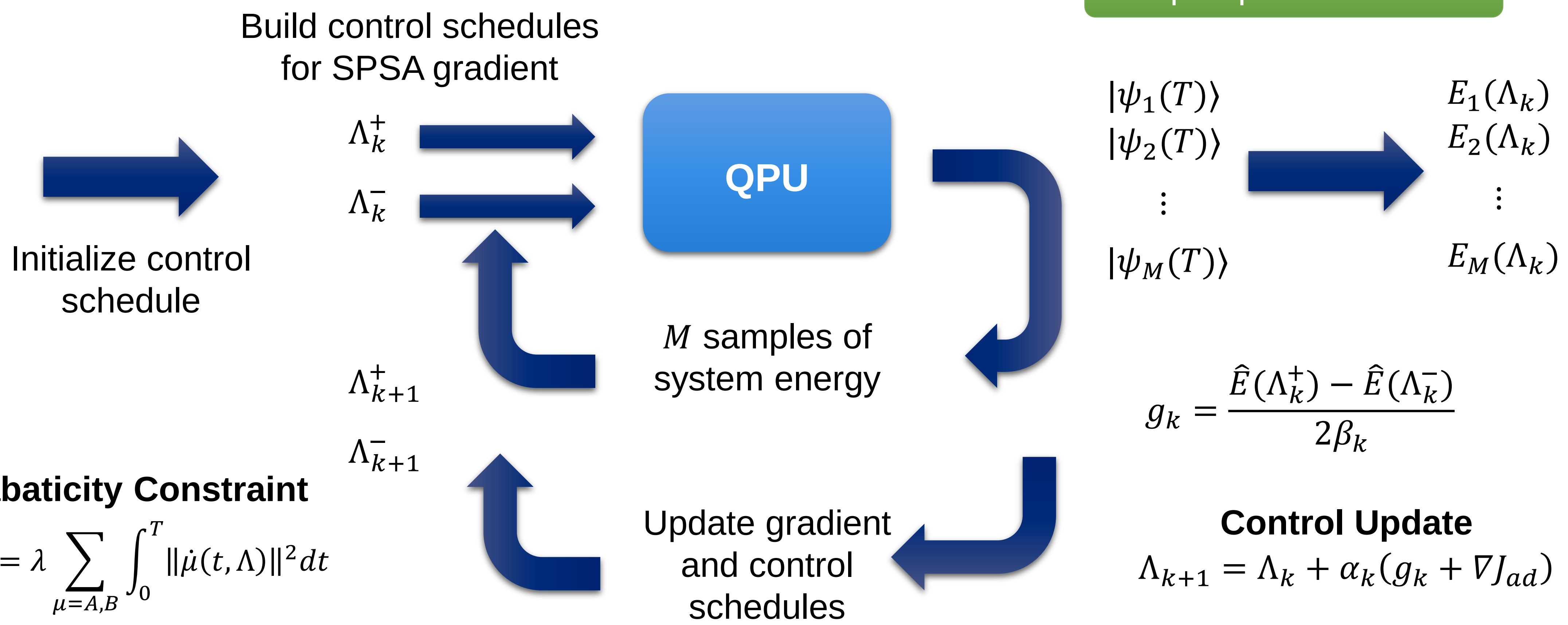
Req. Experiments: $2KM$




$$g_k = \frac{\hat{E}(\Lambda_k^+) - \hat{E}(\Lambda_k^-)}{2\beta_k}$$

**Adiabaticity Constraint**

$$J_{ad}(\Lambda) = \lambda \sum_{\mu=A,B} \int_0^T \|\dot{\mu}(t,\Lambda)\|^2 dt$$

**Control Update**

$$\Lambda_{k+1} = \Lambda_k + \alpha_k(g_k + \nabla J_{ad})$$

# CLOAQC: Numerical Study

## Grover's Search Problem

**Hamiltonian**

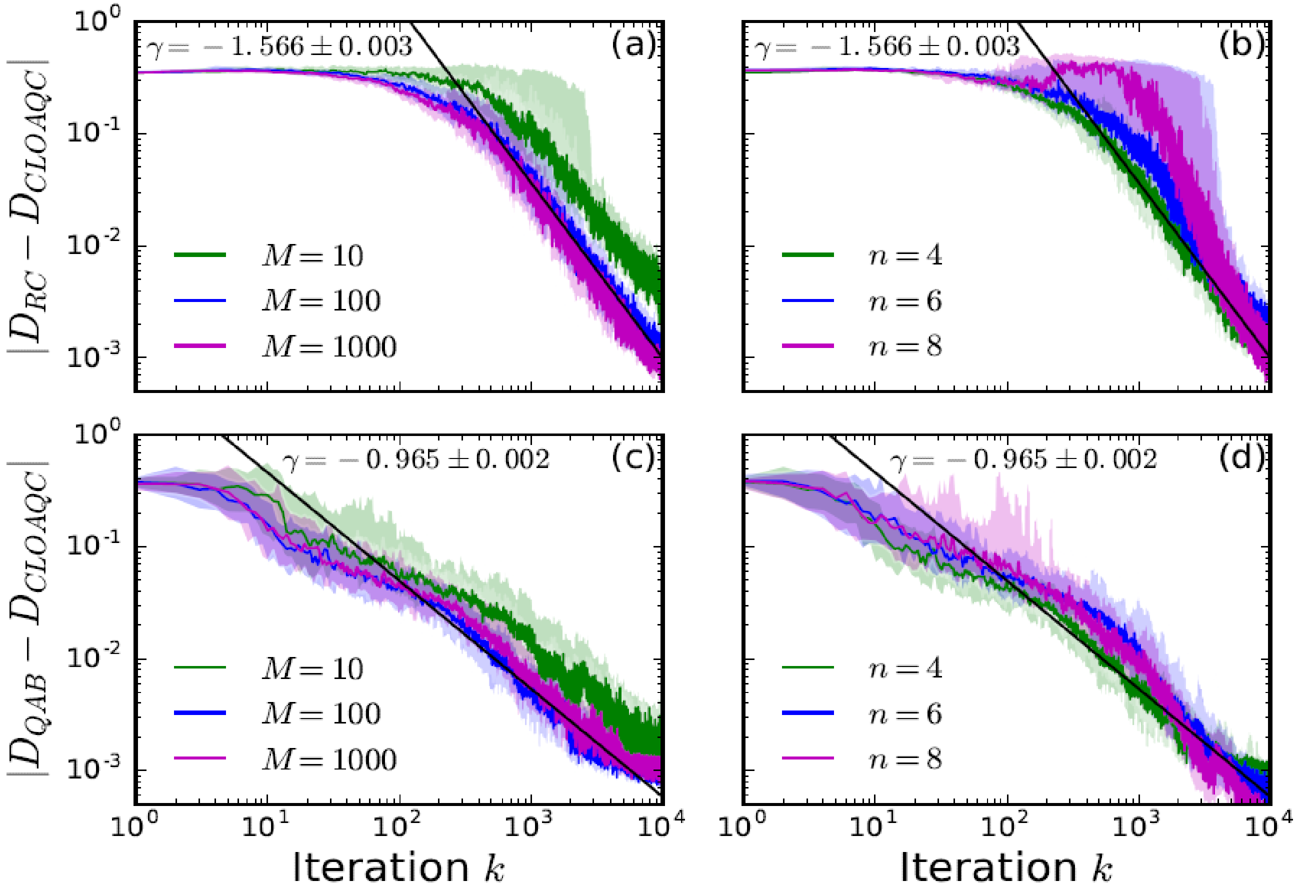
$$H_{ad}(s) = A(s)[I - |+\rangle\langle+|] + \mathrm{B}(s)[I - |m\rangle\langle m|]$$

**Controls**

$$\mathrm{A}(s) = \sum_{i=0}^{d-1} a_i s^i, \mathrm{B}(\mathrm{t}) = \sum_{i=0}^{d-1} b_i s^i$$

**Trace Distance**

$$D = \sqrt{1 - |\langle\Phi_0(1)|\psi(1)\rangle|^2}$$




CLOAQC converges to known LAC solutions

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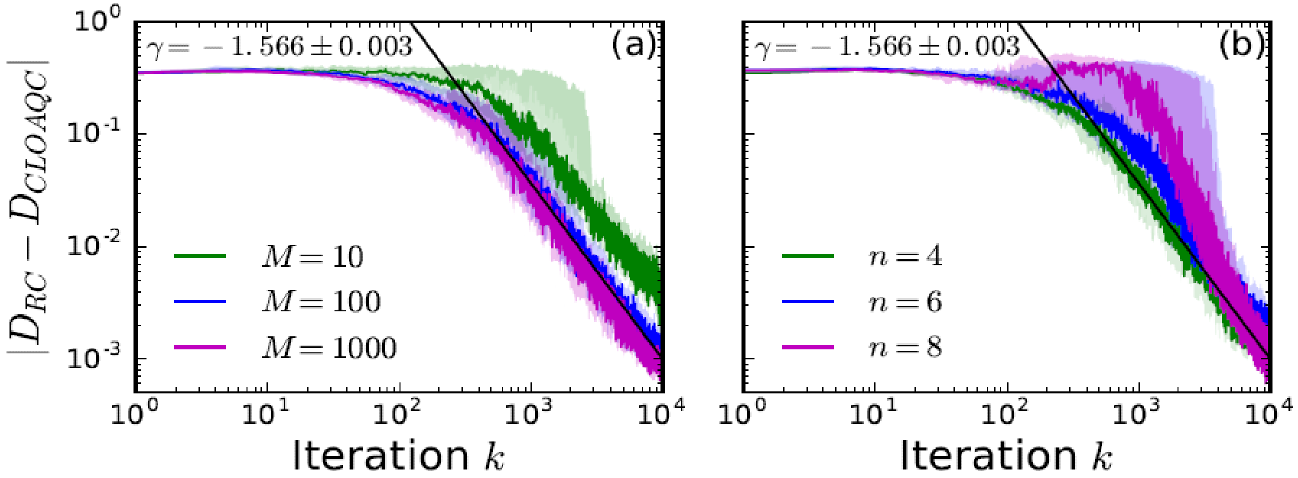
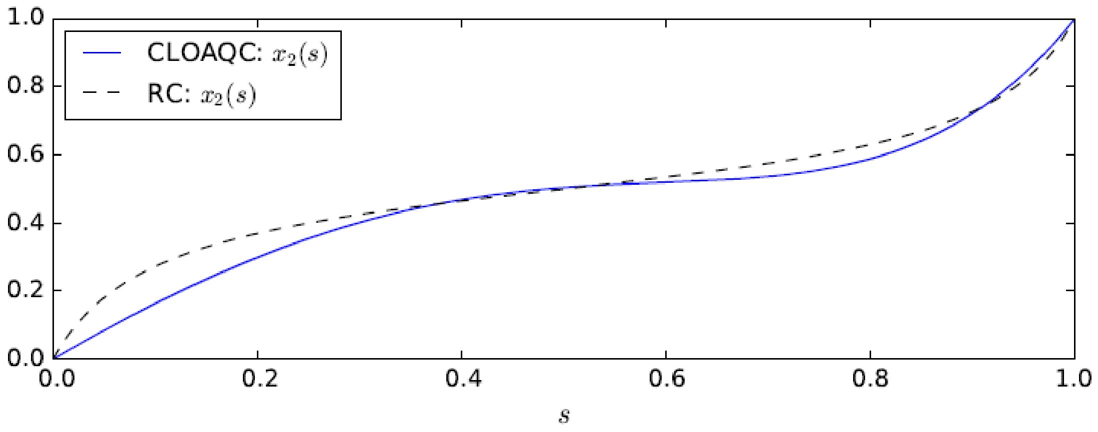


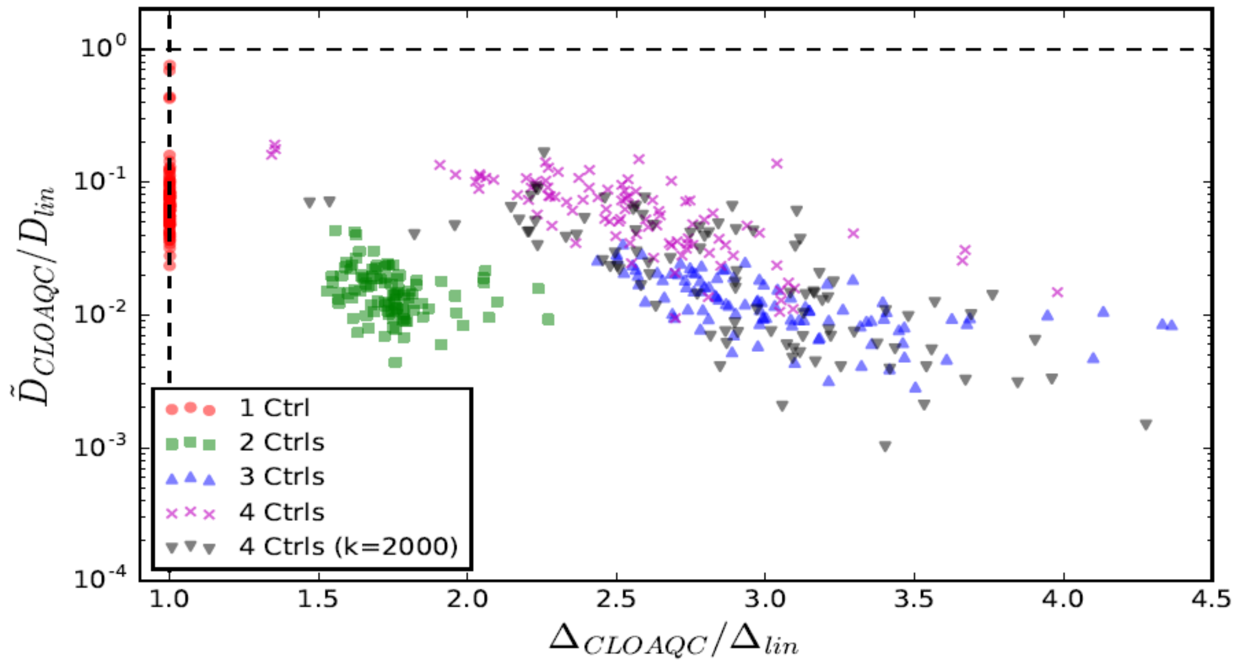
# CLOAQC: Numerical Study

## MAX 2-SAT

**Problem**: Determine maximum number of satisfying assignments for a Boolean formula

$$F[\{x_i\}_{i=1}^{N}] = \underbrace{(x_3 \vee x_1)}_{C_1} \wedge \underbrace{(\neg x_5 \vee x_2)}_{C_2} \wedge \cdots \wedge \underbrace{(x_4 \vee \neg x_3)}_{C_M}$$

$$H_P = \sum_{k=1}^{M} H_{C_k}, \qquad H_{C_k} = \left(\frac{1 - v_{x_i}^k \sigma_{x_i}^Z}{2}\right)\left(\frac{1 - v_{x_j}^k \sigma_{x_j}^Z}{2}\right)$$




### AQC Hamiltonian

$$H(t) = A_1(t)\sum_i \sigma_i^X + A_2(t)\sum_{i\neq j} \sigma_i^X \sigma_j^X + B_1(t)\sum_i h_i \sigma_i^Z + B_2(t)\sum_{i\neq j} J_{ij}\sigma_i^Z \sigma_j^Z$$

Increasing control DOF leads to improvements in computational accuracy and enhancements in minimum gap

# CLOAQC: Numerical Study

Robustness to Noise

## Grover with unitary control errors

$$H'_{ad}(s) = H_{ad}(s) + H_E(s)$$

$$H_{ad(s)} = A(s)[I - |+\rangle\langle+|] + \mathrm{B}(s)[I - |m\rangle\langle m|]$$

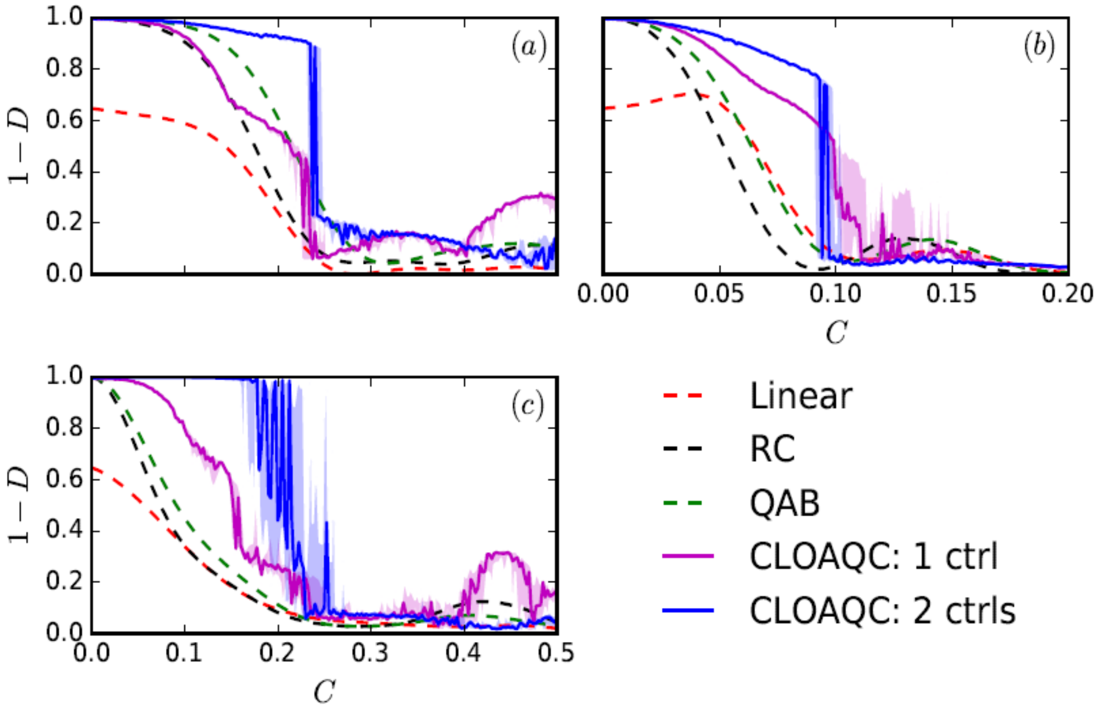
$$H_E(s) = \Gamma(s) \sum_i \hat{m}_i \cdot \vec{\sigma}_i$$

## Error Scenarios

a) $\Gamma(s) = Cs$

b) $\Gamma(s) = C\sin(\pi s)$

c) $\Gamma(s) = \frac{1}{2}\sin(C\pi s)$

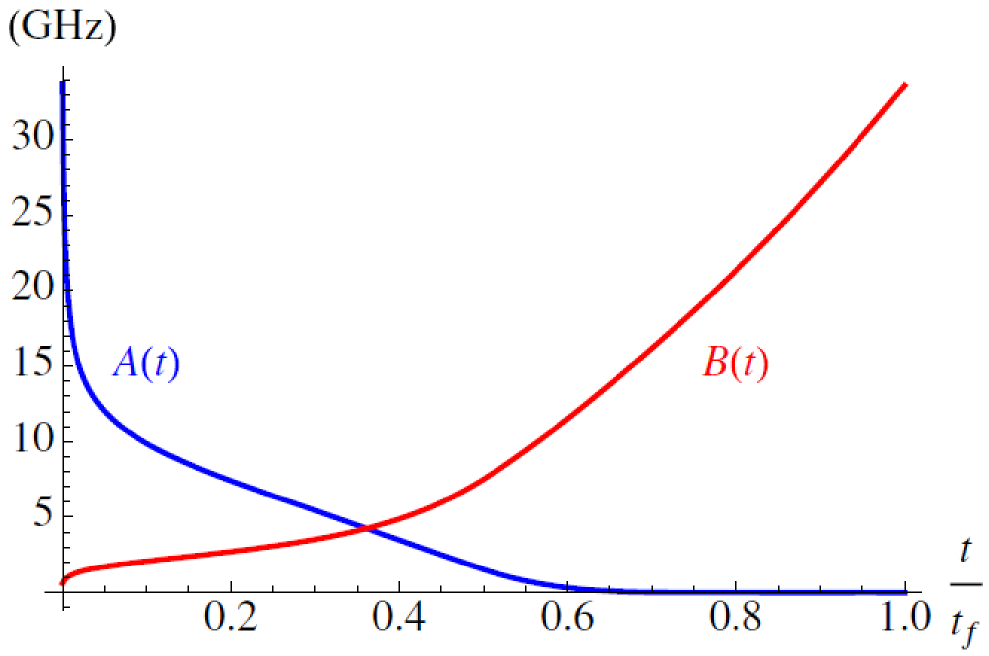



CLOAQC exhibits robustness to sufficiently small and slow-oscillating unitary control errors

# Control Capabilities on the D-Wave QPU

### 2000Q System

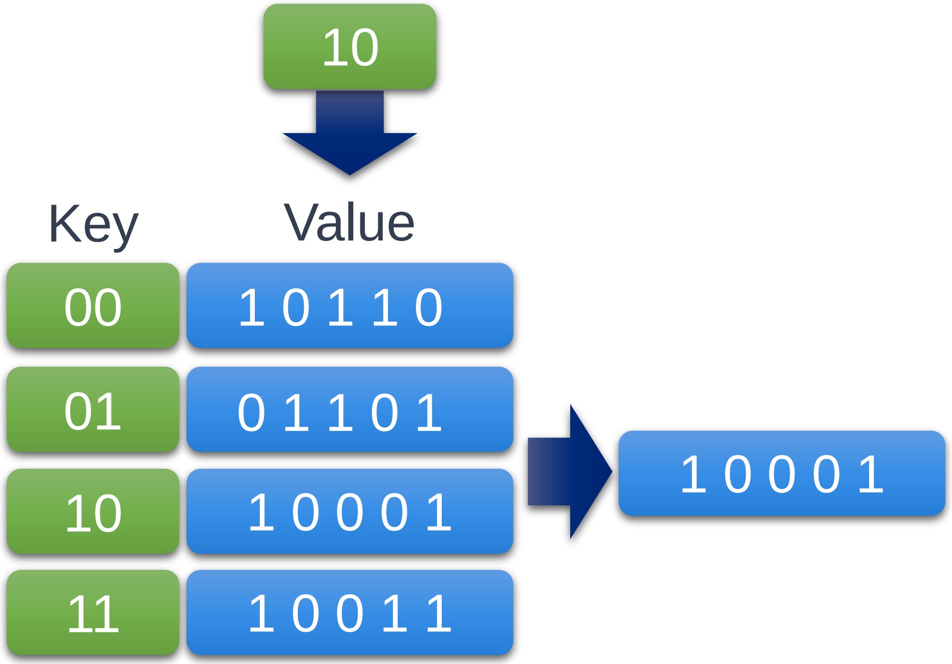
- Allows for *some* control over annealing path
- Path must be monotonic
- New features
  - Pausing
  - Quenching
- Permits experimental testing of CLOAQC!



# Content Addressable Memory Problem

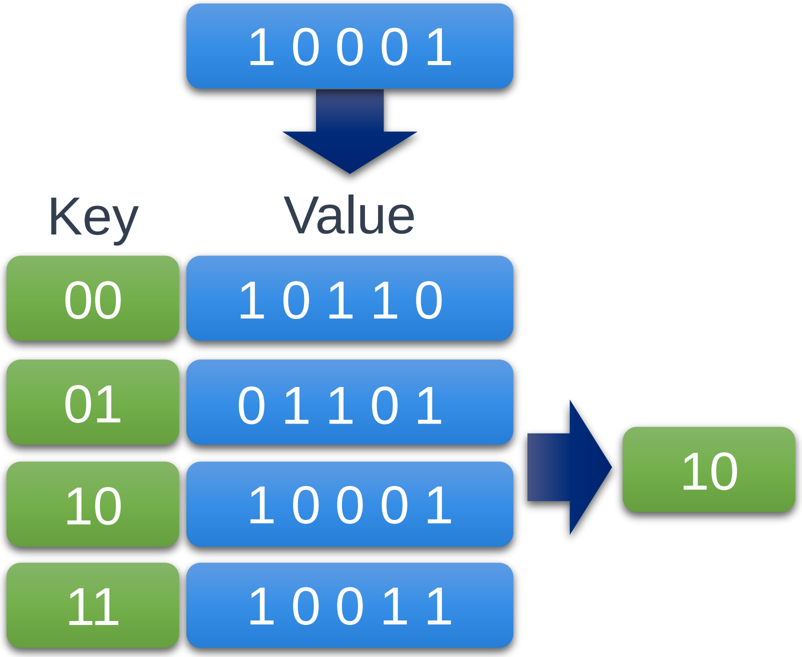
**Traditional Memory**

- Input is address location of the desired content
- Output is the content of the address




**Content Addressable Memory (CAM)**

- Input is content of the stored memory
- Output is the location of the desired content



# Quantum CAM

## Problem Design

Cast CAM problem as an adiabatic quantum optimization problem

Keys: $K = \left[k^{(1)}, k^{(2)}, \dots, k^{(m)}\right]^T$

Values: $V = \left[v^{(1)}, v^{(2)}, \dots, v^{(m)}\right]^T$

## Hamiltonian Description

$$H(t,\theta) = A(t)H_X + B(t)H_\theta$$

$$H_X = -\sum_i^n \sigma_i^X$$

$$H_\theta = -\sum_{i,j}^n w_{ij}\sigma_i^Z\sigma_j^Z - \sum_i^n \theta_i v_i^{(0)}\sigma_i^Z$$

## Hebbs Learning Rule

$$W = \begin{pmatrix} 0 & W_B \\ W_B^T & 0 \end{pmatrix} \qquad W_B = \frac{1}{n}K^T V$$

Maximum Classical Learning Capacity: $C(n) = \frac{n}{2}\log(n)$


H. Seddiqi, T. Humble Frontiers in Phys. 2014
Santa et al. PRA 20017
Schrock et al. Entropy 2017

# QCAM Preliminary Experimental Results

## CLOAQC Convergence Scaling

### Problem Description

- $n = 16$ logical qubits
- # encoded memories: $m = 0.2n$
- 1 $\mu s$ annealing time
- $N$ =1000 annealing runs
- 20 realizations of CLOAQC
- 500 iterations of CLOAQC

### Convergence Scaling

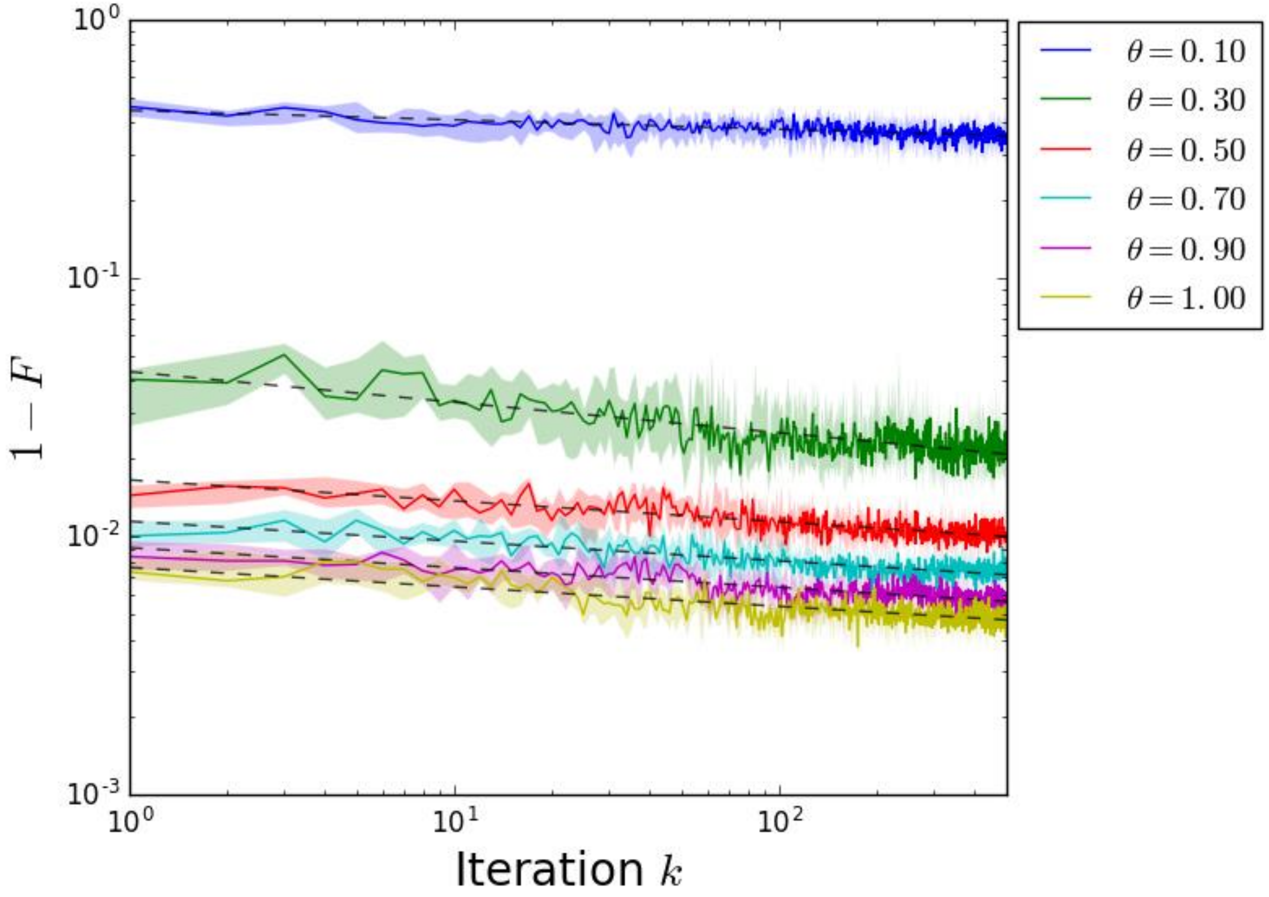
Convergence Rate: $O(k^{\beta})$

| $\theta$ | $\beta$ |
|---|---|
| 0.1 | -0.0364 |
| 0.3 | -0.1182 |
| 0.5 | -0.0808 |
| 0.7 | -0.0766 |
| 0.9 | -0.0768 |
| 1.0 | -0.0752 |

### Fidelity

$$F = \frac{1}{N}\sum_{i}^{N} \delta_{c_i,n}$$

Single Instance Convergence Scaling



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## Fidelity vs. Bias

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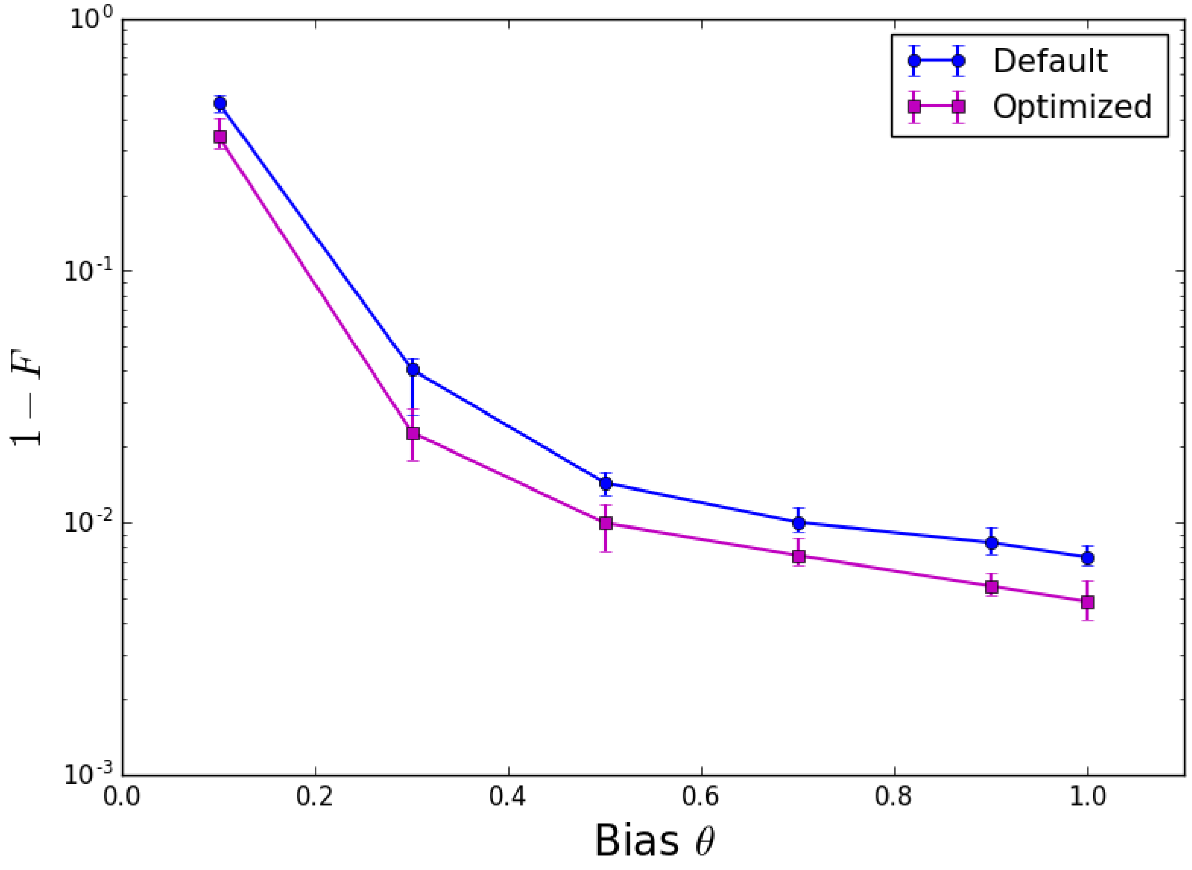
### Fidelity

$$F = \frac{1}{N}\sum_{i}^{N} \delta_{c_i,n}$$

Median Infidelity vs. Bias



# QCAM Preliminary Experimental Results

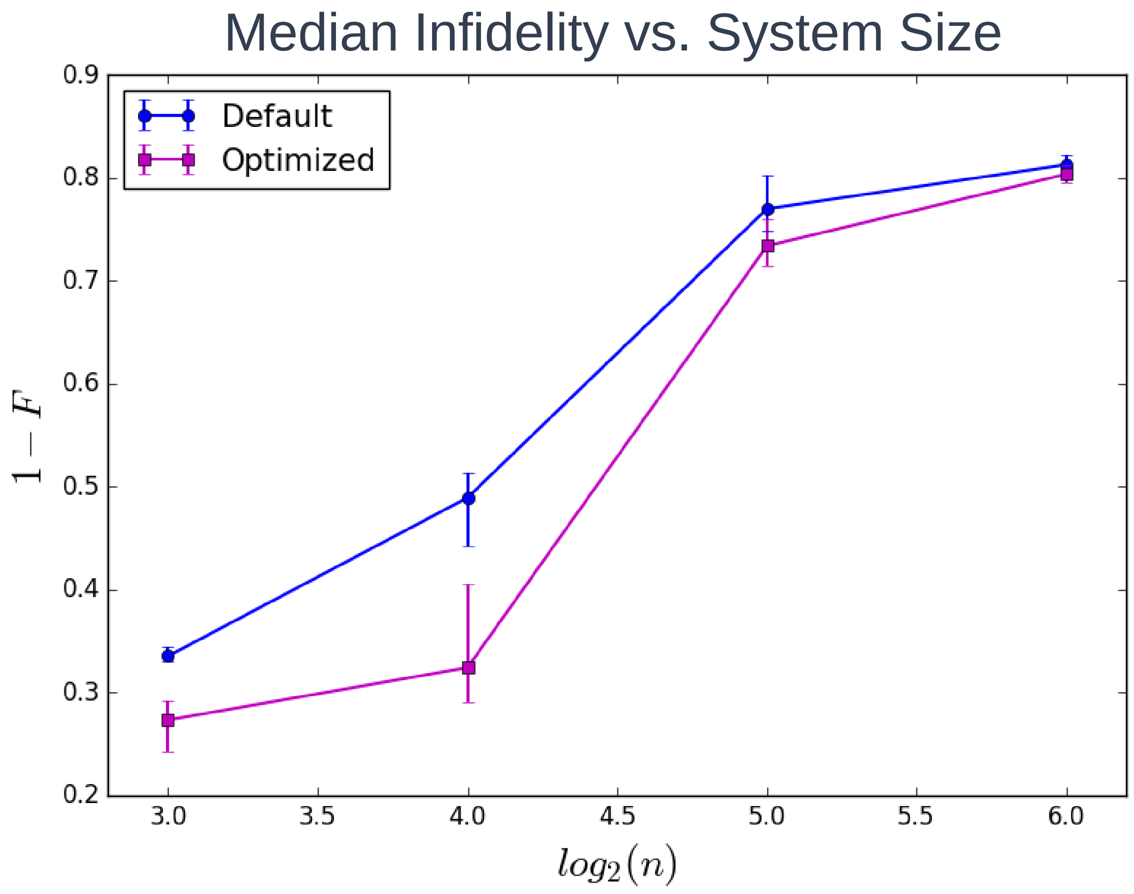
## Fidelity vs. Problem Size

### Problem Description

- Number of logical qubits $n = 8,16,32,64$
- Bias $\theta = 0.1$
- # stored memories: $m = 0.2n$
- 1 $\mu s$ annealing time
- $N$ =1000 annealing runs
- 20 realizations of CLOAQC
- 500 iterations of CLOAQC

### Fidelity

$$F = \frac{1}{N}\sum_{i}^{N} \delta_{c_i,n}$$



# Summary

## Conclusions

- CLOAQC *can* be used to improve computational accuracy of the D-Wave QPU
- Encouraging preliminary results suggest QCAM recall accuracy can be improved by CLOAQC

## Future Work

- Explore benefits of control for QCAM capacity
- Optimizing control with respect to capacity
- Methods for accelerating CLOAQC convergence

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